Why Compound Discounting is Probably Wrong, How to Fix It, and Implications for HEOR

EE433

Charles E. Phelps, PhD; University Professor and Provost Emeritus, University of Rochester, Rochester, NY USA

WHY WE USE COMPOUND DISCOUNTING

It began in medieval banking in 1300s because it was the only math that they could do at the time (simple arithmetic). Full tables published in 1340 CE in "*Handbook of Merchants*." Full details published in 1613 in Witt's "*Arithmeticall [sic] Questions*." Since then, compound discounting is the ***ONLY*** model in use in banking, finance, and... in economics. Samuelson proposed it in 1937 as we now do it the very same way now. It was irresistible for its simplicity of use and it has "stationary" discounting (constant time preferences). However, he denied any meaning of welfare, saying "*…**any connection between utility as discussed here and any welfare concept is disavowed**.*" But everybody bought it!

$\boldsymbol{SAMUELSON}, \mathbf{1937}$: $Value = \int_0^T U(C(t)e^{-\rho t}dt$. The discrete time equivalent is

$Value = \sum_0^T U(C_t)\delta^t$, where $\delta = \left(\frac{1}{1+r}\right)$ and r is the discount rate.

Key features of compound discounting: (1) The marginal rate of substitution between any two adjacent periods is always constant at $\boldsymbol{MRS = \beta}$. **(2)** People **have stable time preferences,** so a comparison of two future periods looks the same now, 2 years from now, 10 years from now and 50 years from now. These are "stationary" time preferences.

Bankers in Medieval Europe used compound discounting because that was the only method that their mathematics—addition, subtraction, multiplication and division— allowed. It would be a galactically amazing coincidence if people's true time preferences happened to match those simple discounting formulas.

PEOPLE'S REAL TIME PREFERENCES ARE HYPERBOLIC

"…when mathematical functions are explicitly fit to… [the]… data, a hyperbolic functional form, which imposes declining discount rates, fits the data better than the exponential functional form, which imposes constant discount rates.".*

"… decision makers exhibit a 'passion for the present' when offered choices between monetary amounts at different dates in the future. That is, the discount rate required to rationalize the choice of money *today* or in the future is extremely high (on the order of hundreds of percent per annum or even thousands of percent), but the discount rate required to rationalize the choice at *two distinct future dates* is relatively low…." **

Functional MRI (fMRI) studies of brain responses to discounting problems verify that humans brains have hyperbolic discounting. Even pigeons have hyperbolic discounting.

"Just noticeable difference" (JND) psychological studies show that people distinguish differences *relative to underlying size*. We measure this by the marginal rate of substitution between two adjacent periods, $MRS = \frac{Y'(t)}{Y'(t+1)}$. In compound discounting, $MRS = \beta$ for all *t*, but JND says that the differences have to grow as *t* grows larger for them to be "noticeable." Compound discounting and JND are irreconcilable.

MARKET INTEREST RATES HAVE A HYPERBOLIC PATTERN

30 year fixed mortgage rates: $\approx \mathbf{3-4}$ ***percent*** per year real

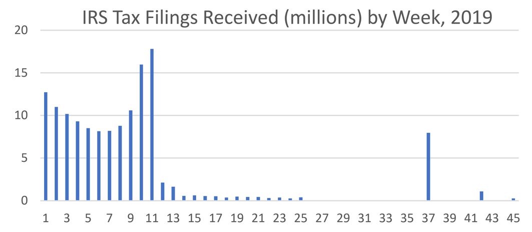
5 year auto loans: $\approx \mathbf{5-6}$ ***percent*** per year real

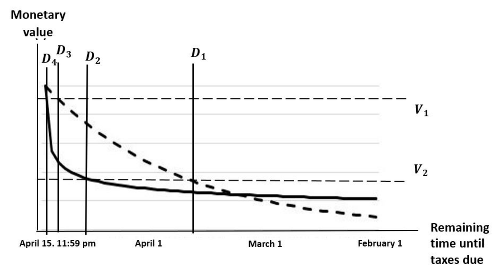
Credit card **monthly** debt: $\approx \mathbf{15-20}$ percent per year real

"Loan Shark" rates: ≈ 20 percent real ***weekly*** (**1000%** per year)

Here, "compound" might refer to leg fractures, not interest rates.

PROCRASTINATION REVEALS HYPERBOLIC DISCOUNTING





* Frederickson, Lowenstein and O'Donoghue," Time Discounting and Time Preference: A Critical Review, *J Econ Lit 2002; 40(2):351-401.*

** Anderson, Harrison, Lau and Rurstrom," "Eliciting Risk and Time Preferences," *Econometrica 2008; 76(3):583-618.*

NUMEROUS *AD HOC* MODELS HAVE BEEN PROPOSED

(1) $D(t) = \frac{1}{t}$; (2) $D(t) = \frac{1}{1+\beta t}$; (3) $D(t) = \frac{1}{(1+\alpha t)^s}$; (4) $D(t) = e^{-\gamma t^C}$

None have any theoretical basis; they just generalize previous proposed functions.

AN ALTERNATIVE MODEL: TIME IS A UTILITY-PRODUCING ASSET

$\boldsymbol{V(C,T) = U(C_t)Y(t)}$ **where both have positive but diminishing marginal utility.**

This is equivalent to standard discounting when $Y(t) = \left[\frac{1}{\rho}\right][1-e^{-\rho t}] = \left[\frac{1}{\rho}\right]\left[\frac{e^{\rho t}-1}{e^{\rho t}}\right]$ since $Y'(t) = e^{-\rho t}$.

This is the widely used exponential utlity (EU). Then, value is $V = \int_0^T U(C)Y'(t)dt = \int_0^T U(C)e^{-\rho t}dt$. **This is exactly equivalent to Samuelson's (1937) formulation.** It has constant absolute risk aversion (CARA), and relative risk aversion is $r^* = \gamma t$, which has strongly increasing relative risk aversion (IRRA). The marginal rate of substitution (MRS) between two adjacent periods is always $e^{-\rho} \approx \left(\frac{1}{1+r}\right) = \beta$.

Benefit cost analysis and cost-effectiveness analysis (BCA/CEA) use an identical formulation, or its discrete time equivalent: $V = \sum_0^T U(C_t)\beta^t$. In health economics, $V = \sum_0^T \beta^t U(C_t)W(H_t)$, where $\beta = \frac{1}{1+r}$, the standard discount modeling. **This comes despite Samuelson's own denial of it as valid welfare measure.**

FIVE PROPOSED CRITERIA FOR SUCCESSORS TO COMPOUND DISCOUNTING

(1) The value at t = 0 is $\boldsymbol{Y(0) = 0}$.
(2) The marginal utility at *t* =0 is $\boldsymbol{Y'(0) = 1}$; (no discount in period zero.).
(3) Like compound discounting, the utility function will have **increasing relative risk aversion** (IRRA).
(4) The discount functions have **finite perpetuity value**.
(5) The **MRS approaches 1 as *t* grows large**; i.e., periods in the distant future are essentially interchangeable.

Criterion (5) is a universal feature of hyperbolic discounting formulas. This criterion is included to incorporate the evidence from the behavioral economics literature that people in fact have hyperbolic discounting.

PREVIOUSLY PROPOSED DISCOUNT FUNCTIONS LEAD TO UTILITY FUNCTIONS

(1) **Inverse of time** (*logarithmic*): $D(t) = \frac{1}{t} \to Y(t) = \ln(t); r^*(t) = 1, MRS = \frac{t}{1+t}, Y'(0) = \infty$. Also, $Y(0)$ is undefined .This function violates four of my criteria; it has an infinitely large perpetuity, the marginal value of time as *t*=0 is infinite; it does not have IRRA and $Y(0)$ is undefined. **It passes only the MRS criterion**.

(2) ***Simple Hyperbolic*** $D(t) = \left(\frac{1}{1+\beta t}\right) \to Y(t) = \left[\frac{1}{\beta}\right]\ln(1+\beta t), r^*(t) = \frac{\beta t}{1+\beta t}; MRS = \frac{1+\beta t}{1+\beta(t+1)}; Y'(0) = 1$; $Y(0) = 0$. **This meets four of my five criteria, but the perpetuity value is infinitely large.**

(3) ***Power Hyperbolic:*** $D(t) = \frac{1}{(1+\alpha t)^s}; Y(t) = \left[\frac{1}{\alpha}\right]\left(\frac{1}{1-s}\right)[1+\alpha t)^{1-s} - 1]; r^* = s\left[\frac{\alpha t}{1+\alpha t}\right]; MRS = \left[\frac{1+\alpha t}{1+\alpha(t+1)}\right]$; $Y'(0) = 1$; $Y(0) = 0$. **Power hyperbolic discounting meets four of my criteria, but the perpetuity is infinitely large** so long as $0 < s < 1$, as required by proponents of this model.

(4) ***Constant Sensitivity:*** $D(t) = e^{-\gamma t^C}; Y(t) = \left[\frac{1}{\gamma C}\right]\left[\frac{t}{t^C}\right]\left[1-e^{-\gamma t^C}\right]; r^*(t) = C\gamma t^C; MRS = \left[\frac{e^{t^C}}{e^{(t+1)^C}}\right]^{\gamma}; Y(0) = 0; Y'(0) = 1$. **This meets four of my five criteria, but the perpetuity value is infinitely large**, since $\left[\frac{t}{t^C}\right]$ grows without bounds as *t* grows large.

ALL OF THESE HAVE $MRS \to 1$ as *t* grows large. Compound discounting fails the MRS test since the $MRS = e^{-\rho}$ for all values of t. All proposed discount functions except compound discounting have infinite perpetuity values. The failure to recognize this pervades the behavioral economics literature.

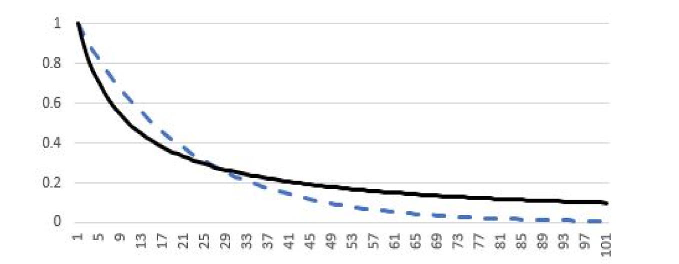
COMMONLY USED ECONOMISTS' UTILITY FUNCTIONS CAN ALSO BE USED AS DISCOUNT FUNCTIONS

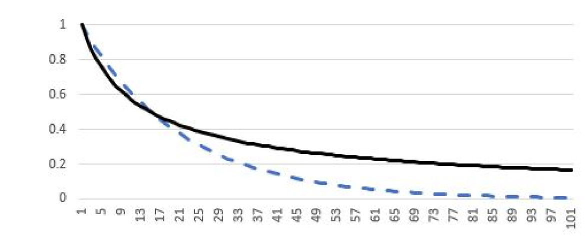
Power Utility; $Y(t) = \left[\frac{1}{\delta}\right]t^{\delta}$; $Y'(t) = t^{\delta-1} = \frac{t^{\delta}}{t}$; $r^*(t) = (1-\delta)$; $MRS = \left[\frac{t}{t+1}\right]^{1-\delta}$. **This violates three of my criteria; it does not have IRRA, the perpetuity value is infinite, $\boldsymbol{Y'(0)}$ is infinite**. It satisfies the MRS criterion and $Y(0) = 0$.

Hyperbolic Absolute Risk Aversion (HARA); $Y(t) = \left[\frac{b^{1-\delta}}{\delta}\right][t+b)^{\delta} - b^{\delta}]; Y'(t) = [b^{1-\delta}[t+b)^{\delta-1}$; $r^*(t) = (1-\delta)\left[\frac{t}{t+b}\right]; MRS = \left[\frac{t+b}{t+b+1}\right]^{1-\delta}$. This has IRRA when $b > 0, Y'(0) = 1, Y(0) = 0$. The MRS approaches 1 as *t* grows large. The perpetuity value is infinite. Thus, **HARA passes four of my five criteria**.

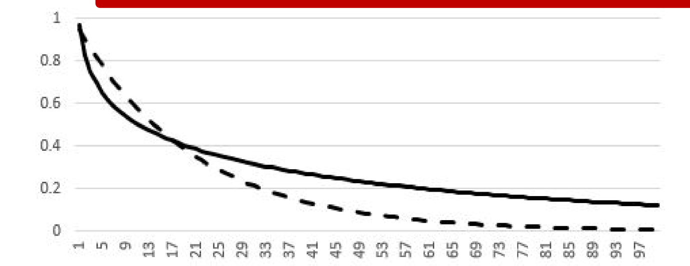
Expo-Power (EP) Utility: $Y(t) = \left[1-e^{-\gamma t^C}\right]\left[\frac{1}{C\gamma}\right] = \left[\frac{1}{C\gamma}\right]\left[\frac{e^{\gamma t^C}-1}{e^{\gamma t^C}}\right]$; $Y'(t) = \frac{t^{C-1}}{e^{\gamma t^C}} = \left[\frac{t^C}{t}\right]\left[\frac{1}{e^{\gamma t^C}}\right]$; $r^*(t) = (1-C) + c\gamma t^C; MRS(t) = \left[\frac{t}{t+1}\right]^{1-C}\left[\frac{e^{t^C}}{e^{t(t+1)^C}}\right]^{\gamma}$. **EP utility passes four of my criteria**, but the marginal utility is explosive as *t* becomes small, since $\left[\frac{t^C}{t}\right] \to \infty$ as $t \to 0$ for $C < 1$ and approaches zero for $C > 1$.

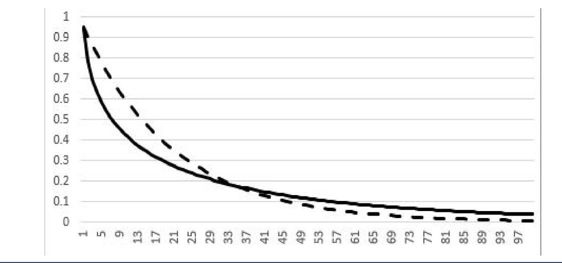
COMPARING HYPERBOLIC AND COMPOUND DISCOUNTING (5%)





Power Hyperbolic utility and EP utility provide considerable flexibility to describe a wide range of time preference profiles.





MY NEW UTILITY FUNCTION SATISFIES ALL FIVE CRITERIA

Generalize EP using HARA, not Power Utility in the exponent, setting $b = 1$.. This "offsets" EP utility by one period. I call this "Expo-HARA" (EH) utility. **This satisfies all of my five criteria,** the only known discounting function that does this.

$Y(t) = \left[1-e^{-\gamma(+1)^C}\right]\left[\frac{e^{\gamma}}{\gamma C}\right] = \left[\frac{e^{\gamma(t+1)^C}-1}{e^{\gamma(t+1)^C}}\right]\left[\frac{e^{\gamma}}{\gamma C}\right] - \frac{[e^{\gamma}-1]}{\gamma C}$; $Y'(t) = [e^{\gamma}]\frac{(t+1)^{C-1}}{e^{\gamma(t+1)^C}}]$; $Y'(1) = 1$; $Y(0) = 0$; $r^*(t) = \left[\frac{t+1}{t+2}\right][(1-C) + C\gamma(t+1)^C]$; $MRS(t) = \left[\frac{t+1}{t+2}\right]\left[\frac{e^{(t+1)^C}}{e^{(t+2)^C}}\right]^{\gamma}$

NEEDED: STUDIES USING DISCRETE CHOICE EXPERIMENTS

1) Pick a utility function and find the "certainty equivalent. I suggest my new EH function as a good place to start. Alternatives include power-hyperbolic (PH) utility.

2) Ask a number of questions like ""Which would you prefer, $1000 today or $1200 (or some other amount) one year (or some other time) in the future?

3) Estimate the parameters of the utility and discount functions using the answers.

Find the right function!!!

Hint: It's probably not CARA

IMPLICATIONS FOR HEOR

1) **Standard CEA is wrong**; we need to replace β^t with proper $D(t)$. Shifting to hyperbolic discounting will increase the value of treatments with long-term benefits (e.g., for permanent disability arising from diseases like MD, ALS, Alzheimer's, and others, and reduce value of treatments where the benefits occur "in the near future" e.g., vision improvement, skin disorders
2) Standard model of **the value of extending life** (Rosen, 1988) **uses the wrong discount function.** In general, the value of more-distant years rises with hyperbolic discounting, so replacing compound with hyperbolic discounting will increase the value of extending life expectancy in most cases.
3) The way we **derive the value of a Statistical Life Year from estimates of the Value of a Statistical Life use the wrong discount function**, which biases the estimates. Most likely bias: hyperbolic places less weight on high-survival years and more on low-survival years, so denominator will shrink and Value of Statistical Life estimates derived from Value of a Statistical Life Year will increase.

WHAT CAN WE DO AS HEALTH ECONOMISTS???

Estimate *Y'(t)* utility functions using acceptable discount functions. Most promising are "Power Hyperbolic" (but has infinite perpetuity value), Expo-Power (but has explosive *Y'(t)* as *t* approaches 0), or my new Expo-HARA (which meets all criteria). All require two parameters. Test against one-parameter compound discounting at various discount rates.

In all future CEA analyses, not only do sensitivity on "the discount rate," but should also conduct sensitivity analysis on the entire "discount function."