MSR31

Improving Finite-Sample Efficiency in Causal Inference: A Regularized Stabilized IPTW Estimator

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Introduction

- **Problem:** sIPTW estimates of the ATE suffer from variance inflation when propensity scores near 0 or 1 — without a good fix that preserves the full-population estimand
- **Gap:** Trimming and overlap weighting resolve instability but at the cost of shifting the target estimand away from the full-population ATE
- **Method:** Reg-sIPTW was developed by applying a data-adaptive power transformation to weights, tuning parameter α to minimize robust variance under strict covariate balance constraints
- **Result:** Regularization is applied only as needed — preserving ATE consistency while delivering better precision and robustness in finite samples with limited overlap

Backgrounds

- **RCT:** randomization eliminates confounding by design→ no adjustment needed
- Observational studies: non-random treatment assignment → systematic confounding bias
- Propensity Score Matching: widely used but loses sample size, is caliper-sensitive, and underutilizes the propensity score
- **IPTW / sIPTW**: Standard Inverse Probability of Treatment Weighting creates a pseudo-population where treatment ⊥ covariates; stabilized weights (sIPTW = P(A) / P(A|L)) are the current standard
- However, when propensity scores approach 0 or 1, extreme weights destabilize the estimator

- **ATE** — effect on the whole population
- **ATT** — effect on those who received treatment
- **ATO** — effect on the clinically equipoised "overlap" population; naturally avoids positivity issues
- **OW** — Overlap Weighting
- **MSE** — Mean Squared Error

References

The full list of references will be provided by request.

Regularized Stabilized IPTW (Reg-sIPTW)

1. Objective

- **Problem:** sIPTW suffers from high volatility and noise when covariate overlap is limited.
- **Solution:** The Reg-sIPTW method introduces a data-adaptive shrinkage parameter to reduce the influence of extreme weights while preserving the Average Treatment Effect (ATE).

2. Mathematical Formulation

$$w_i^{sIPTW} = \frac{T_i \Pr(T=1)}{\hat{\pi}(X_i)} + \frac{(1-T_i)\Pr(T=0)}{1-\hat{\pi}(X_i)}$$

$$w_i^{Reg}(f) = (w_i^{sIPTW})^{1-f}$$

3. Optimization Process

The optimal regularization factor (f) is selected via a two-step constrained optimization approach:

1) **Baseline Feasibility:** The balance-feasible set is determined using standardized mean difference (SMD) thresholds ($\delta \in \{0.05, 0.10, 0.15, 0.20\}$). Note: If the threshold is not met, the design is structurally infeasible (indicating severe lack of overlap).
2) **Variance Minimization:** Bootstrap resampling is used to find the value of f that minimizes variance while satisfying the balance constraint.

4. Statistical Inference

- **Issue:** Data-adaptive selection of f makes standard model-based errors underestimate sampling variability.
- **Solution:** Bootstrap Resampling is used for inference.
- **Implementation:** The entire estimation process including propensity score estimation, selection of f, and treatment effect calculation, is repeated within each replicate to ensure accurate standard errors and confidence intervals.

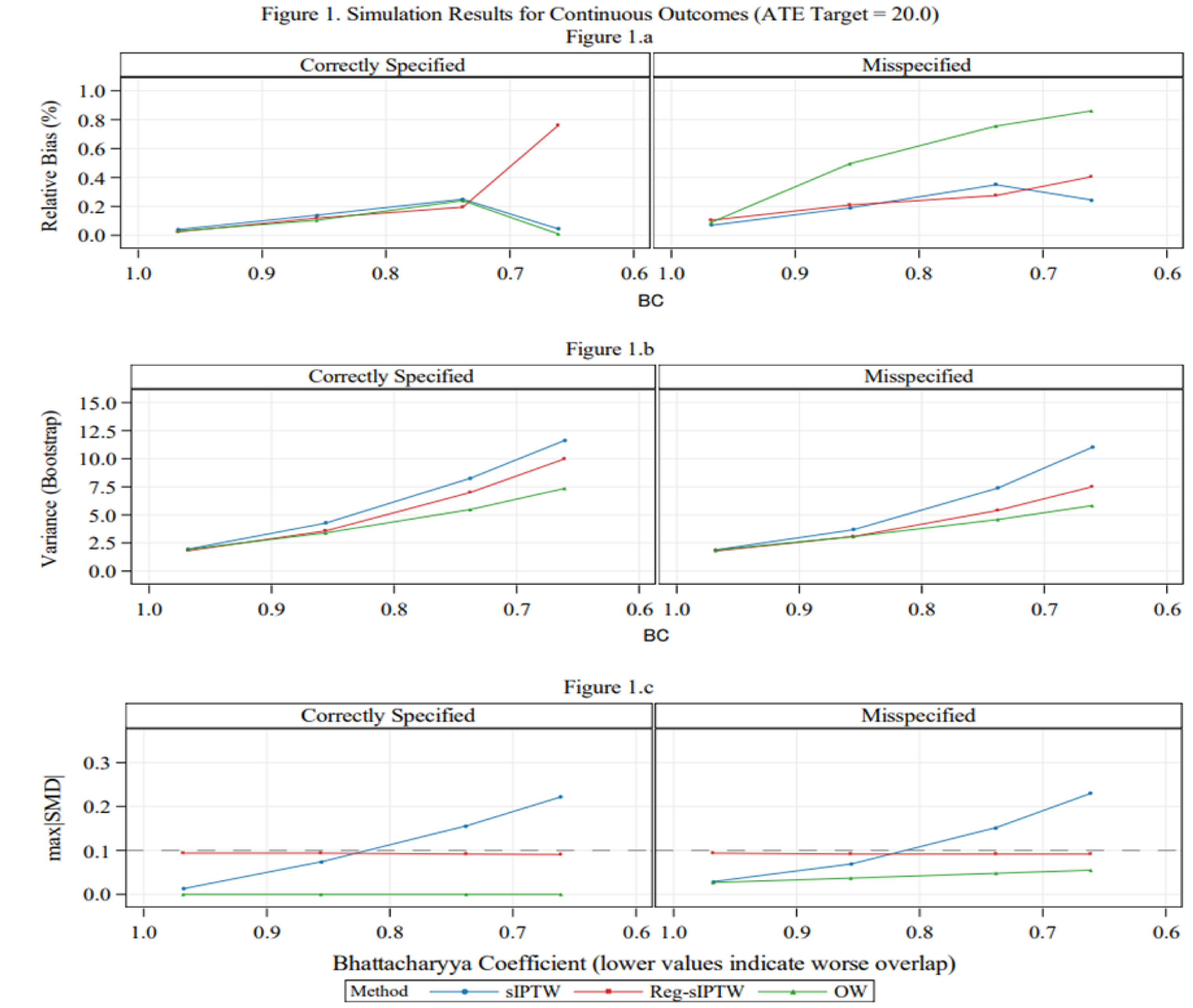
5. Why Reg-sIPTW Has Negligible Bias?

- Hájek form → bias-resistant
- Asymptotically unbiased /consistency, Proved
- Tiny bias, large variance reduction, supported by simulations

Simulation Results

- **Design:** Evaluated sIPTW, Reg-sIPTW, and OW across varying overlap and model misspecification scenarios using 5,000 Monte Carlo replicates.
- **Continuous Outcomes:** Reg-sIPTW reduces variance and improves covariate balance relative to sIPTW while preserving the ATE estimand.
- **Binary Outcomes:** Reg-sIPTW adaptively shrinks extreme weights to control variance and numerical instability in low-overlap settings.
- **Estimand Trade-off:** While OW minimizes variance and achieves exact balance by targeting the ATO, Reg-sIPTW successfully preserves the full-sample ATE.

Figure 1. Simulation Results for Continuous Outcomes (ATE Target = 20.0)



Dashed line in (c): acceptable balance threshold (|SMD| <= 0.10).

Conclusions

- Methodology: Uses adaptive regularization to minimize variance while preserving the ATE.
- Performance: Reduces MSE relative to standard sIPTW without distorting point estimates.
- Robustness: Restores balance and increases effective sample size when facing extreme weights.