

# Rethinking Internal Consistency: Applications of Modern Alternatives

**Patrick Daniele, MSc**

Psychometrician

May 19, 2026

ISPOR 2026

 The world leader in serving science

# Disclosures

- Patrick Daniele is an employee of PPD™ Evidera™ Patient-Centered Research, Thermo Fisher Scientific.

1

## Scenario 1: Violations of Tau-equivalence

- Simulation study comparing Cronbach's alpha and omega total given heterogeneity in factor loadings

2

## Scenario 2: Correlated Errors (Bi-factor Structure)

- Examines the impact of a bi-factor structure on Cronbach's alpha
- Demonstrates omega hierarchical

3

## Scenario 3: Unequal Item Weighting

- Applies coefficient H in the context of scoring beyond simple sum scores

# Scenario 1: Tau-equivalence

- Cronbach's alpha **assumes** a tau-equivalent model:
  - Factor loadings are approximately equal.
  - All items are weighted equally when combined to form the total score.
  - Tau-equivalence implies unidimensionality, but unidimensionality does not necessarily imply tau-equivalence.
- Omega total relaxes the assumption of tau-equivalence.

## Tau-Equivalent<sup>1</sup>

Item	Std. Loading
Q1	0.711
Q2	0.714
Q3	0.716
Q4	0.709
Q5	0.721

## Not Tau-Equivalent<sup>1</sup>

Item	Std. Loading
Q1	0.806
Q2	0.790
Q3	0.725
Q4	0.578
Q5	0.523

# Scenario 1: Tau-equivalence

To demonstrate the impact of violations of tau-equivalence, we conducted a simulation study<sup>1</sup>

- Simulated a 6-item continuous scale based on a prespecified factor structure using the *simulateData* function in lavaan (R).
- Varied factor loadings between 0.2, 0.4, 0.6, and 0.8 to produce 84 distinct combinations.
- Ran 1,000 simulations for each factor loading combination and estimated both alpha and omega.
- Compared alpha and omega with true reliability derived from the factor loadings.

**84 combinations** {

Factor Loading	Reliability
(0.2, 0.2, 0.2, 0.2, 0.2, 0.2)	0.200
(0.4, 0.2, 0.2, 0.2, 0.2, 0.2)	0.256
...	...
(0.8, 0.8, 0.8, 0.8, 0.8, 0.8)	0.914

Inspired by the excellent work here:

1. Edwards AA, Joyner KJ, Schatschneider C. A simulation study on the performance of different reliability estimation methods. *Educ Psychol Meas.* 2021;81(6):1089-1117. doi: 10.1177/0013164421994184. Epub 2021 Feb 15.

PMID: 34565817; PMCID: PMC8451020.

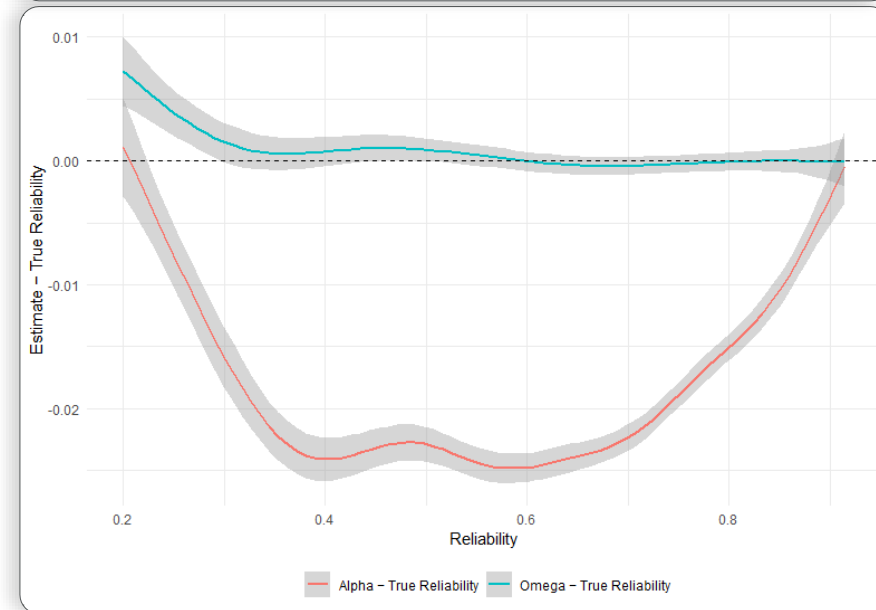
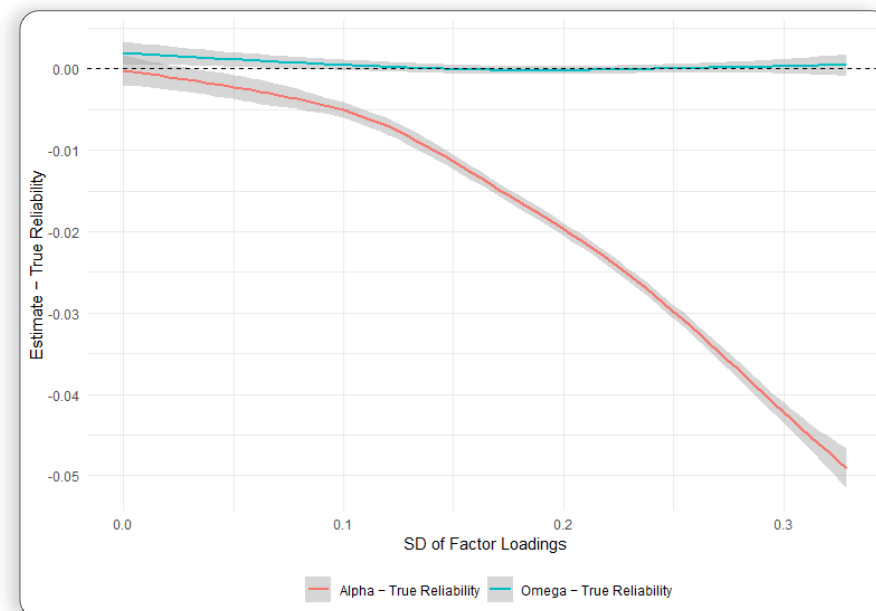
# Scenario 1: Tau-equivalence

## Simulation Results

**Bias in Cronbach's alpha increases with variability in factor loadings.**

- Tends to underestimate true reliability as heterogeneity is introduced.
- Nearly twice as likely as omega to cause errors >10%.
- Average error across all scenarios was 5.0% for Cronbach's alpha and 1.89% for omega when SD of loadings was >0.2.

**Omega is robust to unequal factor loadings.**



# Scenario 2: Correlated Errors

Correlated errors can occur in several ways<sup>1</sup>:



## Content/Item Wording

- Item duplication/redundancy
- Wording effects



## Survey Design

- Order effects
- Reverse coded items



## Model Misspecification

- Omitted factors
- Unmodeled relationships
- Forced orthogonality

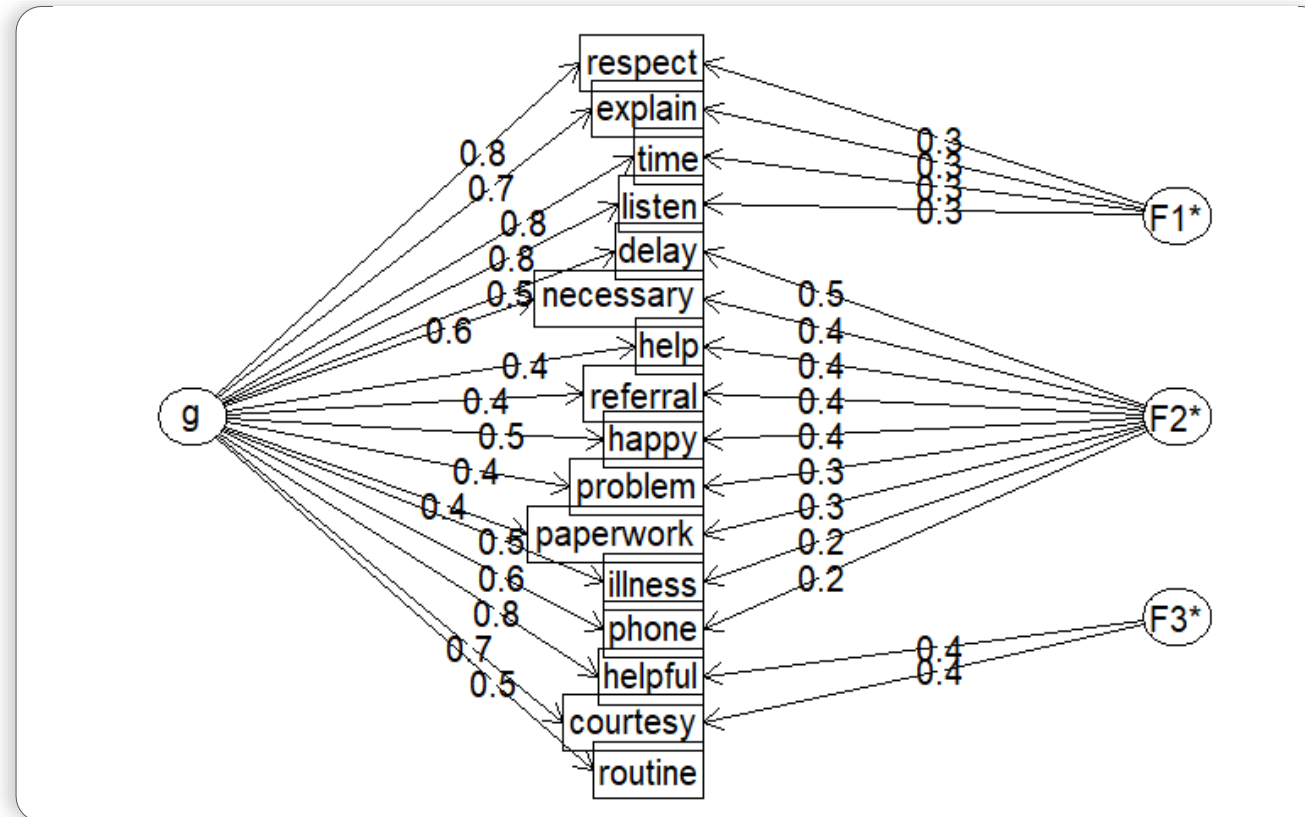


## Data Characteristics

- Longitudinal dependence
- Multi-level design

## Scenario 2: Correlated Errors (Bi-factor)

A bi-factor model structure implies there is a broad “general factor” along with specific subfactors.



## Scenario 2: Correlated Errors (Bi-factor)

- Positively correlated errors can lead to overestimation with Cronbach's alpha (up to ~**20%**).<sup>1</sup>
- Using four datasets with a bi-factor structure, Cronbach's Alpha and Omega Total overestimated reliability relative to Omega Hierarchical between 8% to 45%.

Dataset	Cronbach's Alpha	Omega Total	Omega Hierarchical
Thurstone	0.89	0.93	0.74
Holzinger	0.85	0.89	0.68
Bechtoldt	0.89	0.92	0.61
Reise	0.91	0.92	0.84

1. McNeish D. Thanks coefficient alpha, we'll take it from here. *Psychol Methods*. 2018;23(3):412-433. doi: 10.1037/met0000144. Epub 2017 May 29. PMID: 28557467.

2. Revelle, W. (2024). psych: Procedures for Psychological, Psychometric, and Personality Research. Northwestern University, Evanston, Illinois, USA. <https://CRAN.R-project.org/package=psych>, Version = 2.4.6



## Scenario 3: Unequal Scoring Weights

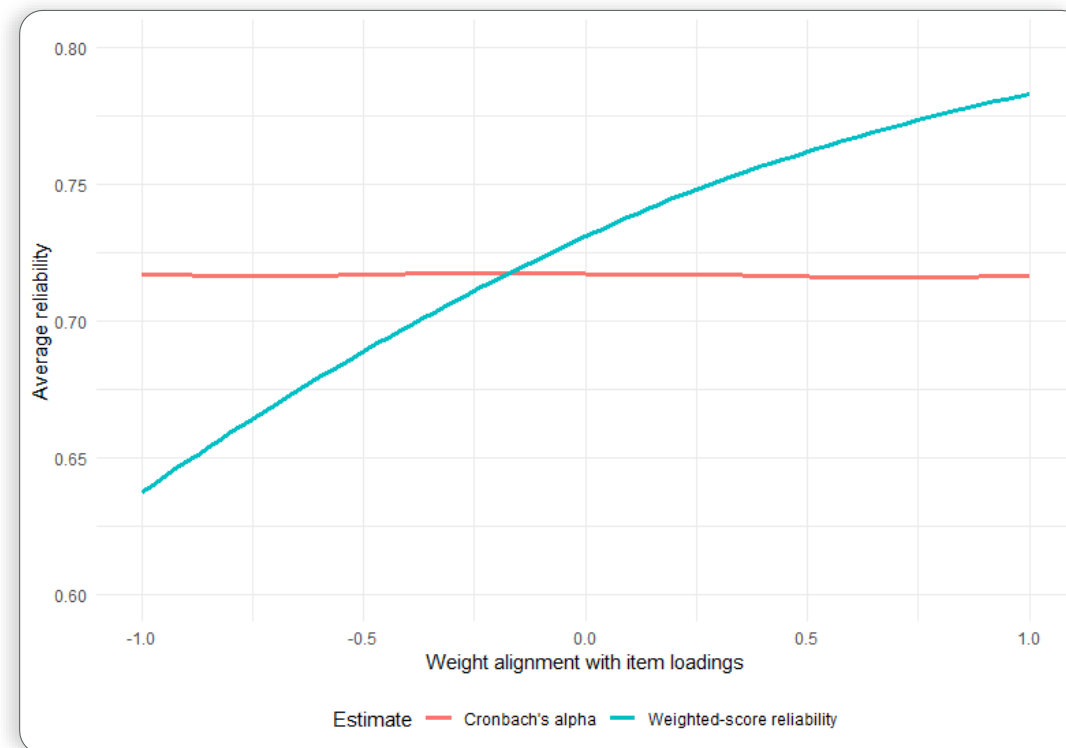
**Measures of reliability should be aligned with the scoring methodology.**

- Cronbach's Alpha is a CTT-based measure of reliability for **total sum score**.
- Coefficient H measures the **optimal weighting** according to the factor loadings.
- Mis-alignment can result in over or underestimation of reliability. To demonstrate, we will explore simulation study wherein vary the weighting of each item according to the factor weights.

# Scenario 3: Unequal Scoring Weights

## Simulation Results

- By varying the scoring algorithm according to the weights of the factor loadings, we can explore the impact of unequal scoring weights on the reliability estimate.
- Cronbach's alpha is constant because the same scoring methodology is applied; however, reliability will be underestimated when scoring weights are treated as equal.
- **Use a measure of reliability that is aligned with the scoring algorithm.**





- ✓ Violations to the assumptions of Cronbach's Alpha can result in bias (either under or over estimation).  
**Check that your instrument does not violate these assumptions.**

---

- ✓ Tau-Equivalence – Are factor loadings equal?

---

- ✓ Unidimensionality – Is the underlying structure a single factor?

---

- ✓ Equal weights – Is the reliability estimate aligned with the score?

---

- ✓ Item Distributions – Are items skewed? Continuous or discrete/ordinal?